Dual Bosonic Thermal Green Function and Fermion Correlators of the Massive Thirring Model at a Finite Temperature

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Abstract

The Euclidian thermal Green function of the two-dimensional (2D) free massless scalar field in coordinate space is written as the real part of a complex analytic function of a variable that conformally maps the infinite strip $-\infty < x < \infty$ ($0 < \tau < \beta$) of the $z = x + i\tau$ (τ: imaginary time) plane into the upper-half-plane. Using this fact and the Cauchy-Riemann conditions, we identify the dual thermal Green function as the imaginary part of that function. Using both the thermal Green function and its dual, we obtain an explicit series expression for the fermionic correlation functions of the massive Thirring model (MTM) at a finite temperature.

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In recent publications, attention has been focused on the thermal Green function of the free massless scalar field in 2D and its relation with the bosonization of the Thirring model, either massive or massless and using both imaginary-time [1, 2] and real-time [3, 4] formalisms. In the former, the thermal correlation functions of bilinears of the fermion field (which are bosonic) of the MTM were derived within the bosonized framework, whereas in the latter, the case of the massless Thirring model is considered.

In the present work, using the imaginary-time formalism, we obtain an explicit series expression for the fermionic thermal correlators of the MTM, thereby filling a gap existing in the subject of bosonization at a finite temperature. In order to do that, we rewrite the thermal Green function in a way that allows us to easily recognize it as the real part of an analytic function. This fact leads us to determine the corresponding dual thermal Green function as the imaginary part of that function, according to the Cauchy-Riemann conditions. This dual thermal Green function turns out to be a key ingredient for the obtainment of the *fermionic* correlators.

The MTM is described by the Lagrangian density

$$\mathcal{L}_{MTM} = i\bar{\psi}\ \partial\!\!\!/\psi - M_0\bar{\psi}\psi - \frac{g}{2}(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi), \tag{1}$$

where ψ is a two-component Dirac fermion field in (1+1)-dimensions. It is well known that it can be mapped, both at $T = 0$ [5] and $T \neq 0$ [1, 2], into the sine-Gordon (SG) theory of a scalar field, whose dynamics is determined by

$$\mathcal{L}_{SG} = \frac{1}{2}\partial_\mu\phi\ \partial^\mu\phi + 2\alpha_0\cos\eta\phi, \tag{2}$$

where the couplings in the two models are related as

$$g = \pi\left(\frac{4\pi}{\eta^2} - 1\right), \qquad\qquad M_0\bar{\psi}\psi = -2\alpha_0\cos\eta\phi. \tag{3}$$

Under this mapping, the two components of the fermion field may be expressed in terms of the SG field as

$$\psi_1(\vec{r}) = \sigma(\vec{r})\mu(\vec{r}), \qquad\qquad \psi_2(\vec{r}) = \sigma^\dagger(\vec{r})\mu(\vec{r}), \tag{4}$$

where $\sigma(\vec{r})$ and $\mu(\vec{r})$ are, respectively, order and disorder fields, satisfying a dual algebra, which can be introduced in the SG theory [6]. These are given by

$$\sigma(x,\tau) = \exp\left\{i\ \frac{\eta}{2}\ \phi(x,\tau)\right\}, \tag{5}$$

$$\mu(x,\tau) = \exp\left\{ i\,\frac{2\pi}{\eta}\int_{-\infty}^{x} dz\,\dot{\phi}(z,\tau)\right\}. \tag{6}$$

Equation (4) coincides with the bosonized expression for the fermion field, first obtained in [7] for the $T=0$ case and also shown to hold at finite temperature in [3, 4].

The Euclidian vacuum functional of the SG theory, for an arbitrary T may be written as the grand-partition function of a classical 2D gas of point charges $\pm\eta$, contained in an infinite strip of width $\beta = 1/k_BT$, interacting through the potential $G_T(\vec{r})$, namely [2]

$$\begin{aligned} \mathcal{Z} = \sum_{m=0}^{\infty}\frac{\alpha_0^m}{m!}\sum_{\{\lambda_i\}_m}\int_0^{\beta}\int_{-\infty}^{\infty}\prod_{i=1}^{m} d\tau_i\,dz_i \\ \times\exp\left\{-\frac{\eta^2}{2}\sum_{i=1}^{m}\lambda_i\sum_{j=1}^{m}\lambda_j G_T(\vec{z}_i-\vec{z}_j)\right\}, \end{aligned} \tag{7}$$

where $\lambda_i = \pm 1$, $\sum_{\{\lambda_i\}_m}$ runs over all possibilities in the set $\{\lambda_1,\ldots,\lambda_m\}$, and $G_T(\vec{r})$ is the thermal Euclidian Green function of the 2D free massless scalar theory in coordinate space ($\vec{r}\equiv(x,\tau)$), which is given by [8]

$$G_T(\vec{r}) = \frac{1}{\beta}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dk}{2\pi}\frac{e^{-i(kx+\omega_n\tau)}}{k^2+\omega_n^2}, \tag{8}$$

with $\omega_n = 2\pi n/\beta$. This has been evaluated in [1] and is given by

$$G_T(\vec{r}) = -\frac{1}{4\pi}\ln\left\{\frac{\mu_0^2\beta^2}{\pi^2}\left[\cosh\left(\frac{2\pi}{\beta}x\right) - \cos\left(\frac{2\pi}{\beta}\tau\right)\right]\right\}. \tag{9}$$

This has also been obtained by using methods of integration on the complex plane [10]. At $T=0$, $G_T(\vec{r})$ reduces to the 2D Coulomb potential and we retrieve the usual mapping onto the Coulomb gas [9].

We may rewrite the above thermal Green function in terms of the new complex variable

$$\zeta(\vec{r}) \equiv \zeta(z) = \frac{\beta}{\pi}\sinh\left(\frac{\pi}{\beta}\,z\right), \tag{10}$$

where $z = x + i\tau$, as

$$G_T(\vec{r}) = \lim_{\mu_0\to 0} -\frac{1}{4\pi}\ln\left[\mu_0^2\zeta(\vec{r})\zeta^*(\vec{r})\right]. \tag{11}$$

At this point let us make a few comments about (11). Firstly, we note that in the zero temperature limit ($T\to 0$, $\beta\to\infty$), we have $\zeta(z)\to z$ and $\zeta^*(z)\to z^*$ and, therefore, we recover the well-known Green function at zero temperature, namely

$$\lim_{\beta\to\infty} G_T(\vec{r};\mu_0) = -\frac{1}{4\pi}\ln\left[\mu_0^2\,zz^*\right] = -\frac{1}{4\pi}\ln\left[\mu_0^2|\vec{r}|^2\right]. \tag{12}$$

Comparing (11) with (12) we can see that the only effect of a finite temperature is to exchange the complex variable z for $\zeta(z)$. Since $\zeta(z)$ is analytic, we conclude that the thermal Green function is obtained from the one at zero temperature by the following conformal mapping [11]: the infinite strip $0 < \tau < \beta$ and $-\infty < x < \infty$ is mapped into the region within the upper-half-ζ-plane.

From Eq. (11) we can also see that the thermal Green function may be written as the real part of an analytic function of the complex variable ζ namely

$$G_T(\vec{r};\mu_0) = \mathrm{Re}\left[\mathcal{F}(\zeta)\right] = \frac{1}{2}\left[\mathcal{F}(\zeta) + \mathcal{F}^*(\zeta)\right], \tag{13}$$

where $\mathcal{F}(\zeta) \equiv -(1/2\pi)\ln\left[\mu_0\zeta(\vec{r})\right]$.

The imaginary part of $\mathcal{F}(\zeta)$ may be written as

$$\begin{aligned}\tilde{G}_T(\vec{r}) \equiv \mathrm{Im}\left[\mathcal{F}(\zeta)\right] &= \frac{1}{2i}\left[\mathcal{F}(\zeta) - \mathcal{F}^*(\zeta)\right] \\ &= -\frac{1}{4\pi i}\ln\left[\frac{\zeta(\vec{r})}{\zeta^*(\vec{r})}\right].\end{aligned} \tag{14}$$

Now, from the analyticity of $\mathcal{F}(\zeta)$, then, it follows that its imaginary and real parts must satisfy the Cauchy-Riemann conditions, which are given by

$$\epsilon^{\mu\nu}\partial_\nu G_T = -\partial_\mu \tilde{G}_T, \qquad\qquad \epsilon^{\mu\nu}\partial_\nu \tilde{G}_T = \partial_\mu G_T. \tag{15}$$

This property characterizes $\tilde{G}_T$ as the dual thermal Green function.

Returning to the Euclidian vacuum functional of the SG theory, we see that this may be now written as

$$\begin{aligned}\mathcal{Z} = \lim_{\varepsilon\to 0}\lim_{\mu_0\to 0}\sum_{n=0}^{\infty}\frac{\alpha^{2n}}{(n!)^2}\int_0^\beta\int_{-\infty}^{\infty}\prod_{i=1}^{2n} d\tau_i\, dz_i \\ \times\exp\left\{\frac{\eta^2}{8\pi}\sum_{i\neq j=1}^{2n}\lambda_i\lambda_j\ln\left\{\mu_0^2\left[\zeta(\vec{z}_i-\vec{z}_j)\zeta^*(\vec{z}_i-\vec{z}_j)+|\varepsilon|^2\right]\right\}\right\},\end{aligned} \tag{16}$$

where, in order to obtain (16), we have used the UV-regulated version of $G_T(\vec{r};\mu_0)$, namely

$$G_T(\vec{r};\mu_0,\varepsilon) = -\frac{1}{4\pi}\ln\left\{\mu_0^2\left[\zeta(\vec{r})\zeta^*(\vec{r}) + |\varepsilon|^2\right]\right\}. \tag{17}$$

The renormalized coupling α is related to the one in (2) by $\alpha = \alpha_0\left(\mu_0^2|\varepsilon|^2\right)^{\eta^2/(8\pi)}$ [5].

As in the $T = 0$ case, existence of the $\mu_0 \to 0$ limit imposes the neutrality of the new gas, namely $\sum_{i=1}^m \lambda_i = 0$, because in this case the μ_0-factors are completely canceled.

This implies that the index m appearing in (7) must be even ($m = 2n$) and, therefore, $\sum_{\{\lambda_i\}_m} = (2n)!/(n!)^2$.

We can now determine the fermionic correlators at $T \neq 0$, by using $(4-6)$ along with the gas representation of the vacuum functional, just derived. The bosonization formulae (4) were shown to hold for $T \neq 0$ in the massless theory. Notice however that, since we are making a mass expansion (α-expansion), we are actually considering a sum of fermion correlators in the massless theory. The insertion of σ and μ operators corresponds, respectively, to the introduction of external charges (of magnitude $\eta/2$) and "magnetic" fluxes on the gas [6]. The fermion correlators, then are nothing but the exponential of the interaction energy of the associated classical system. Charges and "magnetic" fluxes interact with their similar, through the thermal Green function $G_T(\vec{r})$, whereas the charge-flux interaction occurs via the dual thermal Green function $\tilde{G}_T(\vec{r})$ [6]. This is the reason why it is crucial to know this function in order to obtain the fermion correlators. In the case of (charge conserving) fermion bilinear correlators, only external charges are inserted into the gas and, therefore, only the thermal Green function $G_T(\vec{r})$ is required.

Following the above considerations and the same procedure employed at $T = 0$ [12], we can write the four components of the two-point fermion correlation function of the MTM at finite temperature as

$$
\begin{aligned}
&\langle \psi_{1(2)}(\vec{x}) \psi^{\dagger}_{1(2)}(\vec{y}) \rangle \\
&= \mathcal{Z}^{-1} \sum_{m=0}^{\infty} \frac{\alpha_0^m}{m!} \sum_{\{\lambda_i\}_m} \int_0^{\beta} \int_{-\infty}^{\infty} \prod_{i=1}^{m} d\tau_i \, dz_i \, \exp\left\{ \frac{1}{2} \int d^2 z \, d^2 z' \right. \\
&\times \left[\left(i\eta \sum_{i=1}^{m} \lambda_i \, \delta^2(\vec{z} - \vec{z}_i) + \frac{2\pi}{\eta} \left[\int_{\vec{x}}^{\vec{y}} d\eta_\mu \, \epsilon^{\alpha\mu} \delta^2(\vec{z} - \vec{\eta}) \right] \partial_\alpha^{(z)} \right. \right. \\
&+ (-) i \, \frac{\eta}{2} \left[\delta^2(\vec{z} - \vec{x}) - \delta^2(\vec{z} - \vec{y}) \right] \Big) \left(i\eta \sum_{j=1}^{m} \lambda_j \, \delta^2(\vec{z'} - \vec{z_j}) \right. \\
&+ \frac{2\pi}{\eta} \left[\int_{\vec{x}}^{\vec{y}} d\xi_\nu \, \epsilon^{\beta\nu} \delta^2(\vec{z'} - \vec{\xi}) \right] \partial_\beta^{(z')} + (-) i \, \frac{\eta}{2} \left[\delta^2(\vec{z'} - \vec{x}) - \delta^2(\vec{z'} - \vec{y}) \right] \Big) \\
&\left. \left. \times \, G_T(\vec{z} - \vec{z'}) \right] \right\}
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
&\langle\psi_{1(2)}(\vec{x})\psi^{\dagger}_{2(1)}(\vec{y})\rangle \\
&= \mathcal{Z}^{-1}\sum_{m=0}^{\infty}\frac{\alpha_0^m}{m!}\sum_{\{\lambda_i\}_m}\int_0^{\beta}\int_{-\infty}^{\infty}\prod_{i=1}^{m}d\tau_i\,dz_i\,\exp\left\{\frac{1}{2}\int d^2z\,d^2z'\right. \\
&\quad\times\left[\left(i\eta\sum_{i=1}^{m}\lambda_i\,\delta^2(\vec{z}-\vec{z}_i)+\frac{2\pi}{\eta}\left[\int_{\vec{x}}^{\vec{y}}d\eta_\mu\,\epsilon^{\alpha\mu}\delta^2(\vec{z}-\vec{\eta})\right]\partial_\alpha^{(z)}\right.\right. \\
&\quad\left.+(-)i\,\frac{\eta}{2}\left[\delta^2(\vec{z}-\vec{x})+\delta^2(\vec{z}-\vec{y})\right]\right)\left(i\eta\sum_{j=1}^{m}\lambda_j\,\delta^2(\vec{z'}-\vec{z}_j)\right. \\
&\quad\left.+\frac{2\pi}{\eta}\left[\int_{\vec{x}}^{\vec{y}}d\xi_\nu\,\epsilon^{\beta\nu}\delta^2(\vec{z'}-\vec{\xi})\right]\partial_\beta^{(z')}+(-)i\,\frac{\eta}{2}\left[\delta^2(\vec{z'}-\vec{x})+\delta^2(\vec{z'}-\vec{y})\right]\right) \\
&\quad\left.\left.\times\,G_T(\vec{z}-\vec{z'})\right]\right\}.
\end{aligned}
\tag{19}
$$

After some algebra, we get

$$
\begin{aligned}
&\langle\psi_{1(2)}(\vec{x})\psi^{\dagger}_{1(2)}(\vec{y})\rangle \\
&= \mathcal{Z}^{-1}\sum_{m=0}^{\infty}\frac{\alpha_0^m}{m!}\sum_{\{\lambda_i\}_m}\int_0^{\beta}\int_{-\infty}^{\infty}\prod_{i=1}^{m}d\tau_i\,dz_i\,\exp\left\{\frac{\eta^2}{8\pi}\sum_{i\neq j=1}^{m}\lambda_i\lambda_j G_T(\vec{z}_i-\vec{z}_j)\right. \\
&\quad+(-)\frac{\eta^2}{8\pi}\sum_{i=1}^{m}\lambda_i\,\left[G_T(\vec{z}_i-\vec{x})-G_T(\vec{z}_i-\vec{y})\right]+\frac{\eta^2}{16\pi}\left[G_T(\vec{0})-G_T(\vec{x}-\vec{y})\right] \\
&\quad-\frac{i}{2}\sum_{i=1}^{m}\lambda_i\int_{\vec{x}}^{\vec{y}}d\xi_\nu\epsilon^{\beta\nu}\partial_\beta^{(\xi)}G_T(\vec{z}_i-\vec{\xi})-\frac{\pi}{2\eta^2}\int_{\vec{x}}^{\vec{y}}d\eta_\mu\int_{\vec{x}}^{\vec{y}}d\xi_\nu\,\epsilon^{\alpha\mu}\partial_\alpha^{(\eta)}\epsilon^{\beta\nu}\partial_\beta^{(\xi)} \\
&\quad\left.\times\,G_T(\vec{\eta}-\vec{\xi})-(+)\,\frac{i}{4}\int_{\vec{x}}^{\vec{y}}d\xi_\nu\epsilon^{\beta\nu}\partial_\beta^{(\xi)}\left[G_T(\vec{x}-\vec{\xi})-G_T(\vec{y}-\vec{\xi})\right]\right\}
\end{aligned}
\tag{20}
$$

and

$$
\begin{aligned}
&\langle\psi_{1(2)}(\vec{x})\psi^{\dagger}_{2(1)}(\vec{y})\rangle \\
&= \mathcal{Z}^{-1}\sum_{m=0}^{\infty}\frac{\alpha_0^m}{m!}\sum_{\{\lambda_i\}_m}\int_0^{\beta}\int_{-\infty}^{\infty}\prod_{i=1}^{m}d\tau_i\,dz_i\,\exp\left\{\frac{\eta^2}{8\pi}\sum_{i\neq j=1}^{m}\lambda_i\lambda_j G_T(\vec{z}_i-\vec{z}_j)\right. \\
&\quad+(-)\frac{\eta^2}{8\pi}\sum_{i=1}^{m}\lambda_i\,\left[G_T(\vec{z}_i-\vec{x})+G_T(\vec{z}_i-\vec{y})\right]+\frac{\eta^2}{16\pi}\left[G_T(\vec{0})+G_T(\vec{x}-\vec{y})\right] \\
&\quad-\frac{i}{2}\sum_{i=1}^{m}\lambda_i\int_{\vec{x}}^{\vec{y}}d\xi_\nu\epsilon^{\beta\nu}\partial_\beta^{(\xi)}G_T(\vec{z}_i-\vec{\xi})-\frac{\pi}{2\eta^2}\int_{\vec{x}}^{\vec{y}}d\eta_\mu\int_{\vec{x}}^{\vec{y}}d\xi_\nu\,\epsilon^{\alpha\mu}\partial_\alpha^{(\eta)}\epsilon^{\beta\nu}\partial_\beta^{(\xi)} \\
&\quad\left.\times\,G_T(\vec{\eta}-\vec{\xi})-(+)\,\frac{i}{4}\int_{\vec{x}}^{\vec{y}}d\xi_\nu\epsilon^{\beta\nu}\partial_\beta^{(\xi)}\left[G_T(\vec{x}-\vec{\xi})+G_T(\vec{y}-\vec{\xi})\right]\right\}.
\end{aligned}
\tag{21}
$$

Finally, using (17), (14), and (15), we obtain

$$
\begin{aligned}
&\langle\psi_{1(2)}(\vec{x})\psi^\dagger_{1(2)}(\vec{y})\rangle \\
&= \lim_{\varepsilon\to 0}\lim_{\mu_0\to 0}\mathcal{Z}^{-1}\left[\frac{\zeta(\vec{x}-\vec{y})\zeta(\vec{y}-\vec{x})}{\zeta^*(\vec{x}-\vec{y})\zeta^*(\vec{y}-\vec{x})}\right]^{+(-)\frac{1}{4}}\left[\frac{|\varepsilon|^2}{\zeta(\vec{x}-\vec{y})\zeta^*(\vec{x}-\vec{y})}\right]^{(\frac{\pi}{\eta^2}+\frac{\eta^2}{16\pi})} \\
&\times\sum_{n=0}^{\infty}\frac{\alpha^{2n}}{(n!)^2}\int_0^\beta\int_{-\infty}^{\infty}\prod_{i=1}^{2n}d\tau_i\,dz_i \\
&\times\exp\left\{\frac{\eta^2}{8\pi}\sum_{i\neq j=1}^{2n}\lambda_i\lambda_j\ln\left\{\mu_0^2\left[\zeta(\vec{z}_i-\vec{z}_j)\zeta^*(\vec{z}_i-\vec{z}_j)+|\varepsilon|^2\right]\right\}\right. \\
&\left.+(-)\frac{\eta^2}{8\pi}\sum_{i=1}^{2n}\lambda_i\ln\frac{[\zeta(\vec{z}_i-\vec{x})\zeta^*(\vec{z}_i-\vec{x})+|\varepsilon|^2]}{[\zeta(\vec{z}_i-\vec{y})\zeta^*(\vec{z}_i-\vec{y})+|\varepsilon|^2]}+\frac{1}{2}\sum_{i=1}^{2n}\lambda_i\ln\frac{\zeta(\vec{z}_i-\vec{y})\zeta^*(\vec{z}_i-\vec{x})}{\zeta^*(\vec{z}_i-\vec{y})\zeta(\vec{z}_i-\vec{x})}\right\}
\end{aligned}
\tag{22}
$$

and

$$
\begin{aligned}
&\langle\psi_{1(2)}(\vec{x})\psi^\dagger_{2(1)}(\vec{y})\rangle \\
&= \lim_{\varepsilon\to 0}\lim_{\mu_0\to 0}\mathcal{Z}^{-1}\left[\frac{\zeta(\vec{x}-\vec{y})\zeta^*(\vec{y}-\vec{x})}{\zeta^*(\vec{x}-\vec{y})\zeta(\vec{y}-\vec{x})}\right]^{+(-)\frac{1}{4}}\left[\mu_0^2|\varepsilon|^2\right]^{(\frac{\pi}{\eta^2}+\frac{\eta^2}{16\pi})} \\
&\times\left\{\mu_0^2\left[\zeta(\vec{x}-\vec{y})\zeta^*(\vec{x}-\vec{y})\right]\right\}^{-(\frac{\pi}{\eta^2}-\frac{\eta^2}{16\pi})}\sum_{n=0}^{\infty}\frac{\alpha^{(2n+1)}}{n!(n+1)!}\int_0^\beta\int_{-\infty}^{\infty}\prod_{i=1}^{2n+1}d\tau_i\,dz_i \\
&\times\exp\left\{\frac{\eta^2}{8\pi}\sum_{i\neq j=1}^{2n+1}\lambda_i\lambda_j\ln\left\{\mu_0^2\left[\zeta(\vec{z}_i-\vec{z}_j)\zeta^*(\vec{z}_i-\vec{z}_j)+|\varepsilon|^2\right]\right\}+(-)\frac{\eta^2}{8\pi}\sum_{i=1}^{2n+1}\lambda_i\right. \\
&\times\ln\left\{\left\{\mu_0^2\left[\zeta(\vec{z}_i-\vec{x})\zeta^*(\vec{z}_i-\vec{x})+|\varepsilon|^2\right]\right\}\left\{\mu_0^2\left[\zeta(\vec{z}_i-\vec{y})\zeta^*(\vec{z}_i-\vec{y})+|\varepsilon|^2\right]\right\}\right\} \\
&\left.+\frac{1}{2}\sum_{i=1}^{2n+1}\lambda_i\ln\frac{\zeta(\vec{z}_i-\vec{y})\zeta^*(\vec{z}_i-\vec{x})}{\zeta^*(\vec{z}_i-\vec{y})\zeta(\vec{z}_i-\vec{x})}\right\}.
\end{aligned}
\tag{23}
$$

In (22), we still have neutrality of the gas with n positive and n negative λ_i's. In (23), on the other hand, we have n positive and $n+1$ negative λ_i's for $\langle\psi_1\psi_2^\dagger\rangle$. Conversely, for $\langle\psi_2\psi_1^\dagger\rangle$, we have n negative and $n+1$ positive λ_i's. Notice, however, that overall neutrality is still preserved in all functions if we consider the external charges.

Let us finally remark that in the $T\to 0$ limit the above correlators reduce to the corresponding functions of the zero temperature theory [12]. Also, observe that in the limit $\alpha\to 0$ we recover the thermal fermion correlators of the massless Thirring model. These are given by the pre-factor multiplying the sum in (22). In the case of the chirality violating functions (23), the massless limit yields zero as it should.

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